Geometry, Mechanics, and Control in Action for the Falling Cat

岩井敏洋
京都大学　名誉教授

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名古屋大学
情報学研究科棟８階８０２号室

Abstract

猫の宙返りの要諦は、多粒子系の全角運動量ゼロの条件を深く理解するところにある。そのためには、空間の回転対称性に由来する多粒子系のバンドル構造をとらえて、接続の理論を援用し、さらにそれを基に力学系を記述することが肝要である。多粒子系から剛体系の力学に拡張することにより、全角運動量ゼロの条件を上手く取り込んで、猫のモデルが出来上がる。制御入力を上手く設計して、猫の力学系モデルに宙返りをさせるには、制御入力が力学系を整形する役割をはたすという視点が役に立つ。

For an N-particle system, there are $N-1$ mass-weighted vectors (Jacobi vectors), in terms of which the condition for the vanishing of the total angular momentum is expressed as

$$\sum \boldsymbol{r}_i \times \frac{d\boldsymbol{r}_i}{dt} = 0. \tag{1}$$

Physically speaking, any motion satisfying this equation is viewed as vibrational motion, as the total angular momentum of the particle system vanishes. Vibrational motions (or solutions to (1)) prove to exhibit remarkable geometric properties and explain a reason why vibrations can give rise to effective rotations. In addition, a further question arises as to whether the following partial differential equations may have solutions or not,

$$\sum \boldsymbol{r}_i \times d\boldsymbol{r}_i = 0. \tag{2}$$

In other words, the question is stated as follows: Are there $(3N-6)$-dimensional surfaces satisfying the above equations in $\mathbb{R}^{3N-3}$? This equation has been of central interest in the study on the possibility of the separation of vibration from rotation. However, Eq. (2) is not integrable. The quantity associated with the non-integrability has a geometric meaning. Since the falling cat is allowed to make vibrational motions only in the air, a way to the falling cat problem must go through questions as to Eqs. (1) and (2) without reference to gravitational force.

As is well known, cats always can land on their legs when launched in the air. If cats are not given a non-vanishing angular moment at an initial instant, they cannot rotate during their motion, and the motion they can make in the air is vibration only. However, cats accomplish a turn without rotation, when landing on their legs. As is alluded above, in order to solve this apparent mystery, one needs to thoroughly understand rotations and vibrations. The connection theory in Differential geometry can provide rigorous definitions of rotation and vibration for a many particle system. Deformable bodies of cats are not easy to mechanically treat. A feasible way to approach the question about the falling cat is to start with many-body systems and then proceeds to a system of rigid bodies, and further to jointed rigid bodies, which can approximate the body of a cat. The connection theory is applied to a many-body system to show that vibrational motions of the many-body system can result in rotations, without performing rotational motions. Since the present model of the falling cat is a mechanical object, mechanics of many-body systems and of jointed rigid bodies is to be set up. In order to take into account the fact that cats can make their bodies deformed, torque inputs should be applied as control inputs. Further, a suitable control is to be designed so that the model cat may make a somersault. The port-controlled Hamiltonian method will be adopted for the jointed rigid bodies to perform a turn and to halt the motion at the instance of landing. A brief review of control systems will be given through simple examples to explain the role of control inputs.